QEC

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What Errors?

- Decoherence
  - Qubit interacts with an environmental factor
  - Superposition collapses
- Crosstalk (interactions) between adjacent qubits
- Gate produces wrong qubit value(s)
- Measurement produces wrong value

What Errors?

- X-flip (bit flip error)
  - Like classical
  - $|\psi> = a|0> + b|1>$ becomes
    $|\psi> = a|1> + b|0>$

- Z-flip (phase flip error)
  - Doesn’t change probabilities of 0/1
  - $|\psi> = a|0> + b|1>$ becomes
    $|\psi> = a|0> - b|1>$

Complications

- No-cloning theorem
  Classical ECC uses duplication of values
- X and Z flips
- Superposition collapse at measurement

Evolution of QEC

- 1995: Shor code
- 1996: Steane code
- 1997: Topological codes
- 2002: Surface code
- 2006: Bacon-Shor code; 3D Color code
- 2009: Hypergraph-product codes
- 2013: Homological graph
- 2020: Flag-qubit code
- ...

Classical ECC

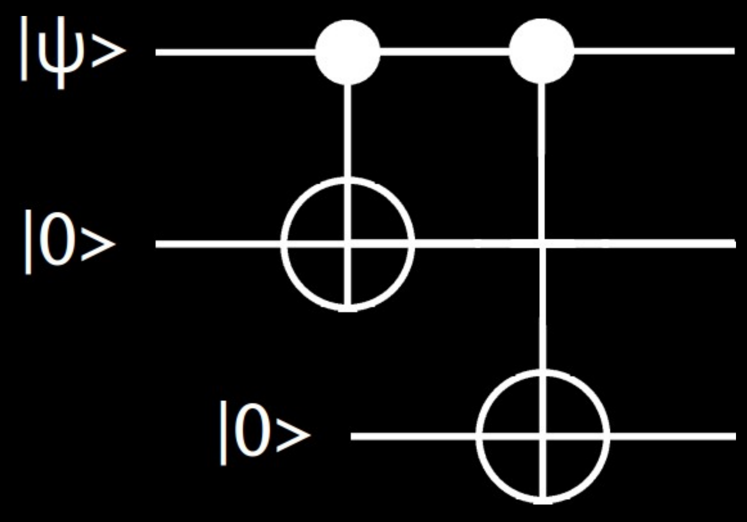
- Three copies of the bit:
  - $0 \rightarrow 000$, $1 \rightarrow 111$
  - Correct by majority vote
- Distance between code words C_i and C_j is $\delta(C_i, C_j) = 2t + 1$
  - Can detect up to δ-1 errors
  - Can correct up to t, i.e., $\lfloor (\delta-1) / 2 \rfloor$, errors
  - $[n, k, \delta]$ where $n=3$, $k=1$, $\delta=3$ for the example

Quantum ECC

- An optimal QECC must
  - Detect & correct both X and Z flips
  - Not directly duplicate initial quantum state
  - Not directly measure the qubits

3-qubit Repetition Code

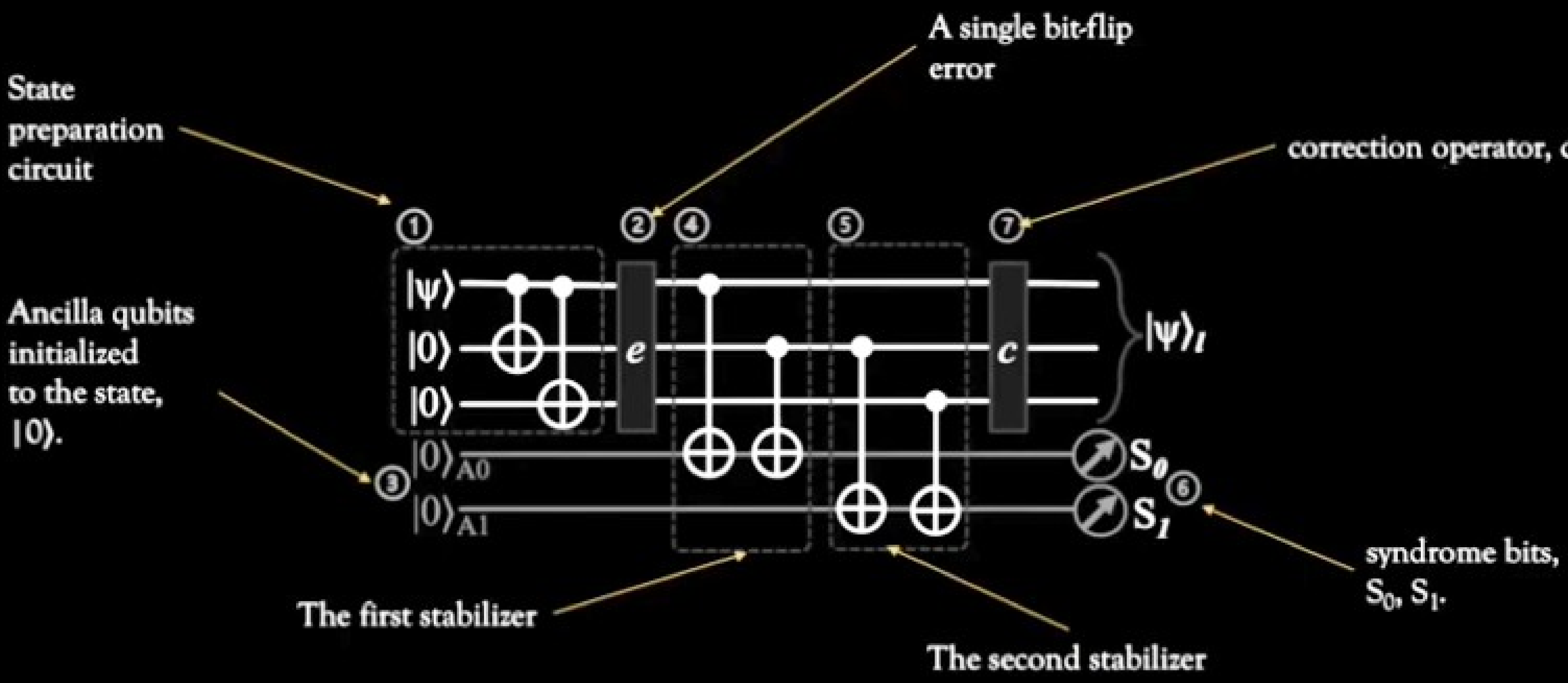
- Three "copies" of the qubit:
  - $|\psi\rangle = a|0\rangle + b|1\rangle$ becomes
    $|\psi\rangle_{logical} = a|000\rangle + b|111\rangle$
  - Implemented using two ancilla with CNOT:



Stabilizers & Commuting

- Stabilizer codes check the parity of two or more qubits using CNOT gates to produce +1 or -1
  - Even parity: +1
  - Odd parity: -1
- Commuting if $[o_i, o_j]=0$
  i.e., $(o_i \otimes o_j)=(o_j \otimes o_i)$
- Anticommuting if $[o_i, o_j] \neq 0$
  i.e., $(o_i \otimes o_j)=-(o_j \otimes o_i)$

3-qubit Repetition Code



3-qubit Repetition Code

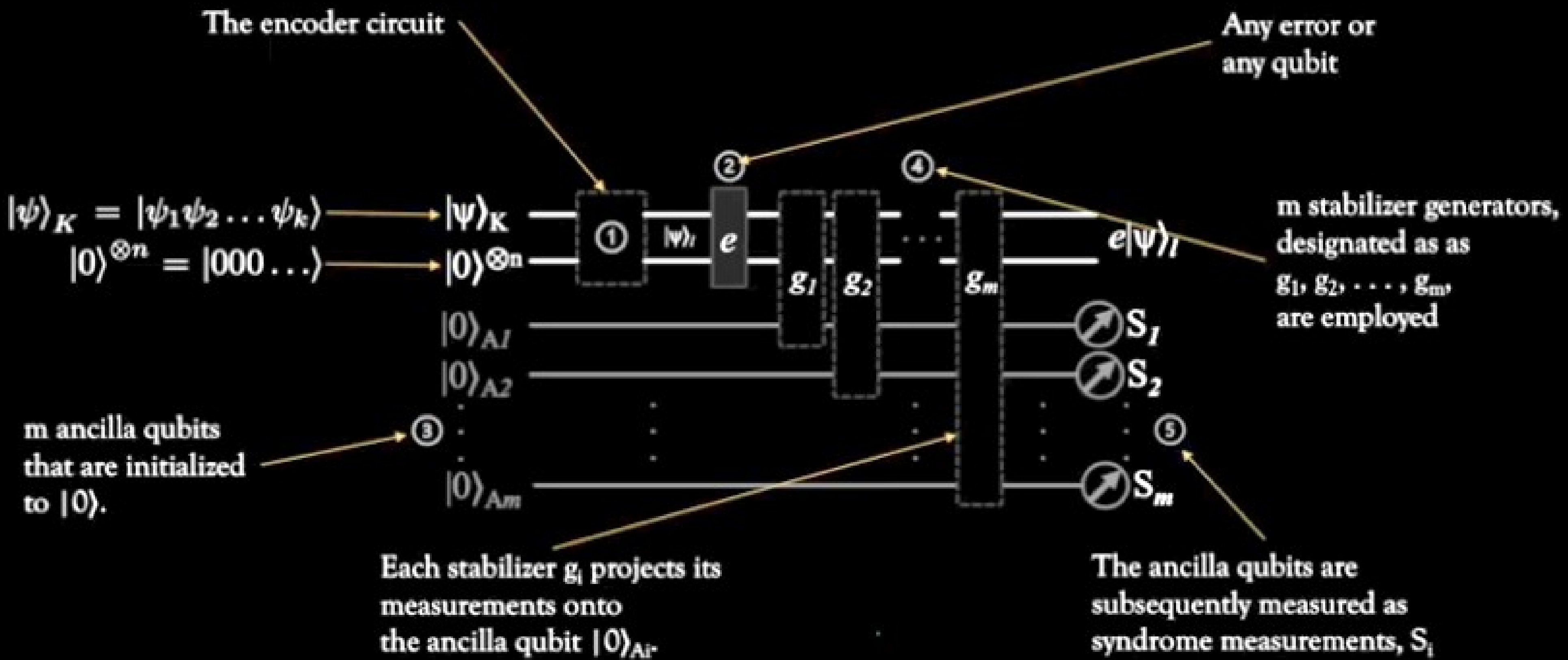
Error	$Z \otimes Z \otimes I$	$Z \otimes I \otimes Z$	$I \otimes Z \otimes Z$
$I \otimes I \otimes I$	+1	+1	+1
$X \otimes I \otimes I$	−1	−1	+1
$I \otimes X \otimes I$	−1	+1	−1
$I \otimes I \otimes X$	+1	−1	−1
	S_0		S_1

For $|\psi\rangle_1 = \alpha\,|000\rangle + \beta\,|111\rangle$, there are three possible parities :
$(Z \otimes Z \otimes I)$,
$(Z \otimes I \otimes Z)$
$(I \otimes Z \otimes Z)$.

QEC is three-step process:
(i) detection
(ii) decoding
(iii) correction

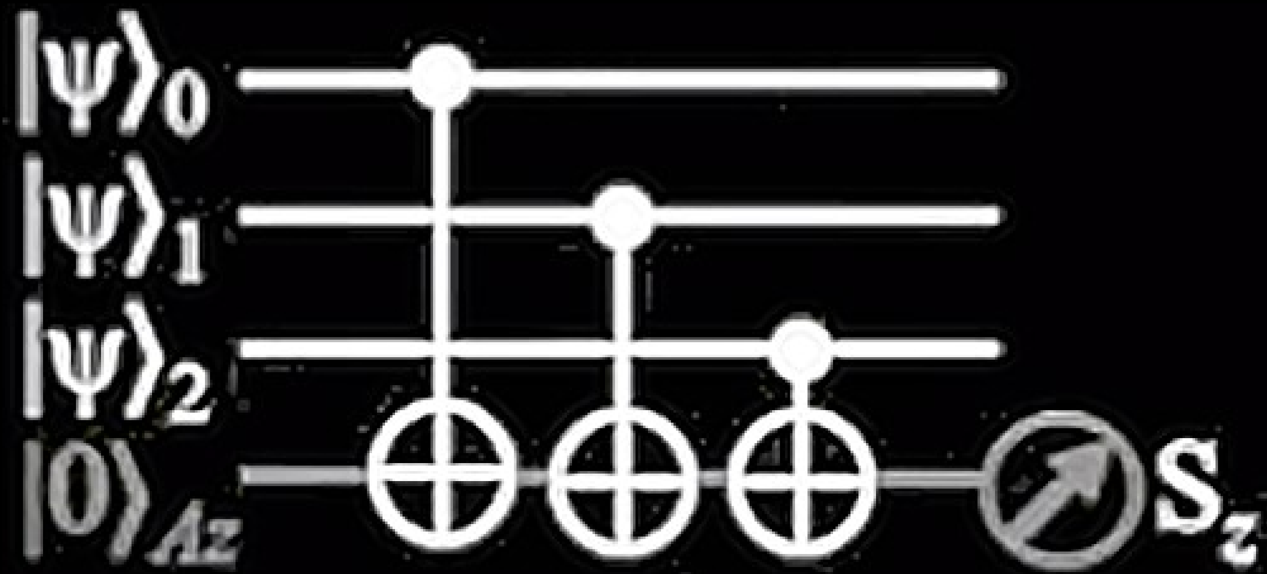
Detection		Deduction		Correction
S_0	S_1	Error Location	Erroneous State	Correction Operator
+1	+1	No error	$\|000\rangle$	III
−1	+1	Qubit 1	$\|100\rangle$	XII
−1	−1	Qubit 2	$\|010\rangle$	IXI
+1	−1	Qubit 3	$\|001\rangle$	IIX

Generalized Stabilizer Circuit



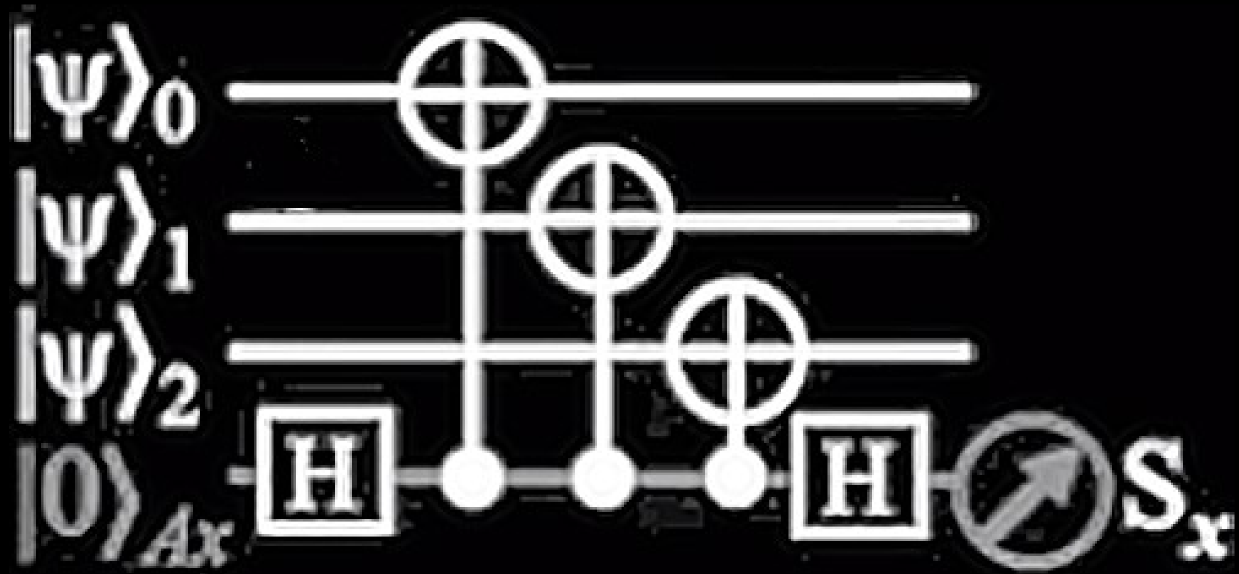
Z Stabilizer Detects X Errors

- On 3 qubits: $Z_0 \otimes Z_1 \otimes Z_2$



X Stabilizer Detects Z Errors

- On 3 qubits: $X_0 \otimes X_1 \otimes X_2$



Topological Codes

- Replication codes directly identify the faulty value, but that doesn't scale
- Topological codes impose a 2D or 3D topology with overlapping detectors
  - Much more scalable!
  - Require analysis of syndromes to identify and correct specific qubit errors

Issues

- Every QEC scheme uses
  - More qubits
  - More operations
  - Wider entanglement (many 2 qubit operators)
- Decoding results can be exponentially hard… so do we use a quantum computer for that?

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  - Like classical

- Z-flip (phase flip error)
  - [[n,k,d]]: n physical bits, encoding k logical bits, with code distance d
  - QECC is [[n,k,d]]: qubits, not bits

What is QEC?

- **Quantum Error Correction** (**QEC**)
  - Encode logical data into physical carriers
  - Transmit or store encoded information
  - Syndrome extraction and recovery (decode)

- Error Correction Codes (ECCs)
  - [[n,k,d]]: n physical bits, encoding k logical bits, with code distance d
  - QECC is [[n,k,d]]: qubits, not bits

Simple QEC References

- A nice overview video:
  https://www.youtube.com/watch?v=oHPwRPeX5ZI

- Wikipedia's not bad either:
  https://en.wikipedia.org/wiki/Quantum_error_correction